IBP & L'Hopital's Review #2

1. $\int x^3 \sin(4x)\,dx$

	u	dv
+	x^3	$\sin(4x)$
−	$3x^2$	$-\frac{1}{4}\cos(4x)$
+	$6x$	$-\frac{1}{16}\sin(4x)$
−	6	$\frac{1}{64}\cos(4x)$
+	0	$\frac{1}{256}\sin(4x)$

$-\frac{1}{4}x^3\cos(4x) + \frac{3}{16}x^2\sin(4x) + \frac{6}{64}\cos(4x) - \frac{6}{256}\sin(4x) + C$

2. $\int e^{3x}\cos(2x)\,dx$

$u = e^{3x}$ $\quad \int dv = \int \cos(2x)\,dx$

$\frac{du}{dx} = 3e^{3x}$ $\quad v = \frac{1}{2}\sin(2x)$

$du = 3e^{3x}dx$

$\boxed{\frac{1}{2}e^{3x}\sin(2x) - \frac{3}{2}}\int e^{3x}\sin(2x)\,dx$

$u = e^{3x}$ $\quad \int dv = \int \sin(2x)\,dx$

$\frac{du}{dx} = 3e^{3x}$ $\quad v = -\frac{1}{2}\cos(2x)$

$du = 3e^{3x}dx$

$-\frac{1}{2}e^{3x}\cos(2x) + \frac{3}{2}\int e^{3x}\cos(2x)\,dx$

$\int e^{3x}\cos(2x)\,dx = \frac{1}{2}e^{3x}\sin(2x) + \frac{3}{4}e^{3x}\cos(2x) - \frac{9}{4}\int e^{3x}\cos(2x)\,dx$

$\frac{13}{4}\int e^{3x}\cos(2x)\,dx = \frac{1}{2}e^{3x}\sin(2x) + \frac{3}{4}e^{3x}\cos(2x)$

$\int e^{3x}\cos(2x)\,dx = \frac{4}{13}\left[\frac{1}{2}e^{3x}\sin(2x) + \frac{3}{4}e^{3x}\cos(2x)\right] + C$

3. $\int x^2 e^{3x}\,dx$

$u = x^2$ $\quad \int dv = \int e^{3x}\,dx$

$\frac{du}{dx} = 2x$ $\quad v = \frac{1}{3}e^{3x}$

$du = 2x\,dx$ $\qquad \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$

$\boxed{\frac{1}{3}x^2e^{3x} - \frac{2}{3}}\int xe^{3x}\,dx$

$u = x$ $\quad \int dv = \int e^{3x}\,dx$

$du = dx$ $\quad v = \frac{1}{3}e^{3x}$

$\frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\,dx$

4. $\int x^2 \arctan(5x^3)\,dx$

$u = \arctan(5x^3)$ $\quad\quad$ $\int dv = \int x^2\,dx$

$\frac{du}{dx} = \frac{15x^2}{1+25x^4}$ $\quad\quad$ $v = \frac{1}{3}x^3$

$\frac{1}{3}x^3 \arctan(5x^3) - \frac{1}{3}\int \frac{15x^5}{1+25x^6}\,dx$

$u = 1+25x^6$ $\quad\quad$ $\frac{1}{3}\int \frac{15\cancel{x^5}}{u} \cdot \frac{du}{150\cancel{x^5}}$

$\frac{du}{dx} = 150x^5$ $\quad\quad$ $\frac{1}{15}\int \frac{1}{u}\,du$

$dx = \frac{du}{150x^5}$ $\quad\quad$ $\frac{1}{15}\ln|u|$

$\frac{1}{15}\ln|1+25x^6|$

$\frac{1}{3}x^3 \arctan(5x^2) - \frac{1}{15}\ln(1+25x^6) + C$

5. $\int x^3 \ln x\,dx$

$u = \ln x$ $\quad\quad$ $\int dv = \int x^3\,dx$

$\frac{du}{dx} = \frac{1}{x}$ $\quad\quad$ $v = \frac{1}{4}x^4$

$du = \frac{1}{x}dx$

$\frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^4 \cdot \frac{1}{x}\,dx$

$\frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\,dx$

$\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

6. $\int 4x^3 e^{2x}\,dx$

	u	dv
+	$4x^3$	e^{2x}
−	$12x^2$	$\frac{1}{2}e^{2x}$
+	$24x$	$\frac{1}{4}e^{2x}$
−	24	$\frac{1}{8}e^{2x}$
+	0	$\frac{1}{16}e^{2x}$

$2x^3e^{2x} - 3x^2e^{2x} + 3xe^{2x} - \frac{3}{2}e^{2x} + C$

7. $\int -5x^2 \ln x \, dx$

$u = \ln x \qquad \int dv = \int -5x^2 \, dx$

$\frac{du}{dx} = \frac{1}{x} \qquad v = -\frac{5}{3}x^3$

$du = \frac{1}{x} dx$

$-\frac{5}{3}x^3 \ln x + \frac{5}{3}\int x^3 \cdot \frac{1}{x} dx$

$-\frac{5}{3}x^3 \ln x + \frac{5}{3}\int x^2 \, dx$

$-\frac{5}{3}x^3 \ln x + \frac{5}{9}x^3 + C$

8. $\int x^2 \cos(3x) \, dx$

$u = x^2 \qquad \int dv = \int \cos(3x) dx$

$\frac{du}{dx} = 2x \qquad v = \frac{1}{3}\sin(3x)$

$du = 2x \, dx$

$\boxed{\frac{1}{3}x^2 \sin(3x) - \frac{2}{3}}\int x \sin(3x) dx$

$u = x \qquad \int dv = \int \sin(3x) dx$

$du = dx \qquad v = -\frac{1}{3}\cos(3x)$

$-\frac{1}{3}x\cos(3x) + \frac{1}{3}\int \cos(3x) dx$

$-\frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x)$

$\frac{1}{3}x^2 \sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + C$

9. $\int 3x^3 \arctan(2x^4) \, dx$

$u = \arctan(2x^4) \qquad \int dv = \int 3x^3 \, dx$

$\frac{du}{dx} = \frac{8x^3}{1+4x^8} \qquad v = \frac{3}{4}x^4$

$\frac{3}{4}x^4 \arctan(2x^4) - \frac{3}{4}\int \frac{8x^7}{1+4x^8} dx$

$u = 1+4x^8 \qquad \frac{3}{4}\int \frac{8\cancel{x^7}}{u} \cdot \frac{du}{32\cancel{x^7}}$

$\frac{du}{dx} = 32x^7 \qquad \frac{3}{16}\int \frac{1}{u} du$

$dx = \frac{du}{32x^7} \qquad \frac{3}{16}\ln|u|$

$\frac{3}{16}\ln|1+4x^8|$

$\frac{3}{4}x^4 \arctan(2x^4) - \frac{3}{16}\ln(1+4x^8) + C$

10. $\int e^{5x}\sin(4x)\,dx$

$u = e^{5x}$ $\qquad \int dv = \int \sin(4x)\,dx$

$\frac{du}{dx} = \frac{1}{5}e^{5x}$ $\qquad v = -\frac{1}{4}\cos(4x)$

$\boxed{-\frac{1}{4}e^{5x}\cos(4x) + \frac{1}{20}}\int e^{5x}\cos(4x)\,dx$

$u = e^{5x}$ $\qquad \int dv = \int \cos(4x)\,dx$

$\frac{du}{dx} = \frac{1}{5}e^{5x}$ $\qquad v = \frac{1}{4}\sin(4x)$

$du = \frac{1}{5}e^{5x}\,dx$

$\frac{1}{4}e^{5x}\sin(4x) - \frac{1}{20}\int e^{5x}\sin(4x)\,dx$

$\int e^{5x}\sin(4x)\,dx = -\frac{1}{4}e^{5x}\cos(4x) + \frac{1}{80}e^{5x}\sin(4x) - \frac{1}{400}\int e^{5x}\sin(4x)\,dx$

$\frac{401}{400}\int e^{5x}\sin(4x)\,dx = -\frac{1}{4}e^{5x}\cos(4x) + \frac{1}{80}e^{5x}\sin(4x)$

$\int e^{5x}\sin(4x)\,dx = \frac{400}{401}\left[-\frac{1}{4}e^{5x}\cos(4x) + \frac{1}{80}e^{5x}\sin(4x)\right] + C$

11. $\lim_{x\to 2} \frac{\sin\left(\frac{\pi x}{2}\right)}{x^2-4} = \frac{0}{0}$

$\lim_{x\to 2} \frac{\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)}{2x} = \frac{-\frac{\pi}{2}}{4} = -\frac{\pi}{8}$

12. $\lim_{x\to\infty} x^{\frac{1}{x}}$

$y = \lim_{x\to\infty} x^{\frac{1}{x}}$

$\ln y = \lim_{x\to\infty} \ln x^{1/x}$

$\ln y = \lim_{x\to\infty} \frac{1}{x}\cdot \ln x = \frac{\infty}{\infty}$

$\ln y = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$\ln y = 0$

$y = e^0$

$y = 1$

13. $\lim_{x \to 1} \frac{\ln(x)}{x} = \frac{0}{1} = 0$

14. $\lim_{x \to -\infty} x e^{x} = -\infty \cdot 0$

$\lim_{x \to -\infty} \frac{x}{e^{-x}} = \frac{-\infty}{\infty}$

$\lim_{x \to -\infty} \frac{1}{-e^{-x}} = 0$